Exercise 45

Find the derivative of the function.

$$y = \cos\sqrt{\sin(\tan \pi x)}$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned}
y' = \frac{dy}{dx} &= \frac{d}{dx}\left[\cos\sqrt{\sin(\tan \pi x)}\right] \\
&= \left[-\sin\sqrt{\sin(\tan \pi x)}\right] \cdot \frac{d}{dx}\left[\sqrt{\sin(\tan \pi x)}\right] \\
&= \left[-\sin\sqrt{\sin(\tan \pi x)}\right] \cdot \frac{1}{2}\left[\sin(\tan \pi x)\right]^{-1/2} \cdot \frac{d}{dx}\left[\sin(\tan \pi x)\right] \\
&= \left[-\sin\sqrt{\sin(\tan \pi x)}\right] \cdot \frac{1}{2}\left[\sin(\tan \pi x)\right]^{-1/2} \cdot \left[\cos(\tan \pi x)\right] \cdot \frac{d}{dx}(\tan \pi x) \\
&= \left[-\sin\sqrt{\sin(\tan \pi x)}\right] \cdot \frac{1}{2}\left[\sin(\tan \pi x)\right]^{-1/2} \cdot \left[\cos(\tan \pi x)\right] \cdot (\sec^2 \pi x) \cdot \frac{d}{dx}(\pi x) \\
&= \left[-\sin\sqrt{\sin(\tan \pi x)}\right] \cdot \frac{1}{2}\left[\sin(\tan \pi x)\right]^{-1/2} \cdot \left[\cos(\tan \pi x)\right] \cdot (\sec^2 \pi x) \cdot (\pi) \\
&= -\frac{\pi}{2}\frac{\sin\sqrt{\sin(\tan \pi x)}\cos(\tan \pi x)\sec^2 \pi x}{\sqrt{\sin(\tan \pi x)}}
\end{aligned}$$